PD. Dr. M. Amann

Winter semester
2016/2017
University of Bonn

Seminar on Differential Geometry
Lie groups and their representations

Lie groups combine differential geometry with group theory, as they are smooth manifolds with smooth groups multiplication and inversion. Lie groups and constructions arising from them (like quotients) form highly remarkable classes of manifolds in Riemannian geometry. They also play a key role in many areas of mathematics and motivate further studies in, for example, algebraic and geometric topology or (geometric) group theory.

In this seminar we want to study compact Lie groups and their representations. Our focus will lie on the so-called classical groups which appear as matrix groups.

Time and place

The seminar takes place on Thursdays, 2-4 p.m., Room 1.008.

Previous knowledge

This seminar aims towards Bachelor and Master students with basic knowledge in differential geometry.

Chapters 1 and 2 of [Ada69] are left for self-study.

Schedule

20/10: (1) Basic representation theory I. (Hernández Cácerez).
Basic definitions and constructions, direct sums, tensor products.
Literature: [Ada69, Chapter 3, p. 22–p. 30, Prop. 3.8]

27/10: (2) Basic representation theory II. (Conradi).
Homomorphisms and the Hom-functor.
Literature: [Ada69, Chapter 3, p. 30, Def. 3.9–p. 39]

03/11: (3) Basic representation theory III. (Beaumont).
Irreducible and virtual representations.
Literature: [Ada69, Chapter 3, p. 40–p. 47]

10/11: (4) Basic representation theory IV. (Schreyer).
Characters of representations. The Peter–Weyl theorem.
Literature: [Ada69, Chapter 3, p. 48–p. 57, Lemma 3.44]

17/11: (5) Basic representation theory V. (Tsakanikas).
Class functions and self-conjugate representations.
Literature: [Ada69, Chapter 3, p. 57–p. 66]

24/11: (6) Basic representation theory VI. (Vordosanidze).
Characterisations of real, complex and quaternionic representations.
Literature: [Ada69, Chapter 3, p. 66, Theorem 3.57–p. 78]

01/12: (7) Maximal tori I. (Bürger).
First properties of maximal tori. The maximal tori of the matrix groups.
Literature: [Ada69, Chapter 4, p. 78, Theorem 3.57–p. 89]

08/12: (8) Maximal tori II. (Beck).
Every element is contained in a maximal torus. (Proof uses Lefschetz fixed-point formula.)
Literature: [Ada69, Chapter 4, p. 89, Theorem 4.21–p. 96]

15/12: (9) Maximal tori and Stiefel diagrams. (Gentille).
Regular and singular point, the Stiefel diagrams of matrix groups.
Literature: [Ada69, Chapters 4,5, p. 97, Examples 5.4–p. 105]

22/12: (10) Stiefel diagrams. (Papanntouis).
The action of the Weyl group on Weyl chambers.
Literature: [Ada69, Chapter 5, p. 105, Theorem 5.5–p. 116]

12/01: (11) Stiefel diagrams II. (de la Torre).
Weights and roots. Positive, negative and simple roots. Dynkin diagrams.
Literature: [Ada69, Chapter 5, p. 116, Discourse 5.18–p. 127]

19/01: (12) Stiefel diagrams III. (Wu).
Fundamental groups of (classical) Lie groups.
Literature: [Ada69, Chapter 5, p. 127, Discourse 5.46–p. 141]

26/01: (13) Representation theory. (Danica).
Weyl integration formula. Special characters.
Literature: [Ada69, Chapter 6, p. 142–p. 153, Proposition 6.19]

02/02: (14) Representation theory II. ().
Discussing the representation ring. Show that it is a polynomial ring.
Literature: [Ada69, Chapter 6, p. 153, Theorem 6.20–p. 164]

09/02: (15) Representations of the classical Lie groups. ().
Computations of the representation rings of the classical groups.
Literature: [Ada69, Chapter 7, p. 165–p. 176]

Some hints for preparing seminar talks

- It is highly advised to start to prepare your seminar talk soon enough, so that potential problems and uncertanties can be removed at an early stage.
- Check what was done in previous talks, what is needed for subsequent ones, and synchronise with your colleagues.
- A clear, well-elaborated, and, especially, well-presented talk is expected. You should be able to stress the main results, definitions and concepts of your talk, in particular, you should be able to put them and their importance into the context of the whole seminar. Proofs should be explained in clarity and with precision. Proof ideas must be visible.
- A seminar talk must illustrate, elaborate and exemplify the material from the book, not only copy it.
- Make sure that your blackboard notes are well-structured, easily readable and rather complete.
- Design your seminar talk to last approximately 75 to 80 minutes so that there is enough room for questions and discussion.
- The talk should be helpful and delightful to the whole audience. We want to learn some math; help us to enjoy this!

References

[Ada69] J. Frank Adams, *Lectures on Lie groups*, W. A. Benjamin, Inc., New York-Amsterdam, 1969. MR0252560

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