Analytic L^2-invariants and ζ-Regulization

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Abstract

The aim of this talk is to give a brief and gentle introduction to ζ-regularized determinants of operators over Hilbert spaces, as well as describing the topological and analytic invariants arising from this construction.
If $\mathcal{A}$ is a finite, tracial von Neumann algebra, $\mathcal{H}$ is a *finitely generated* Hilbert $\mathcal{A}$-module, and A is an $\mathcal{A}$-linear automorphism of $\mathcal{H}$, we can construct a determinant $\mathrm{Det}_\tau(A) \in \mathbb{R}^+$ via so-called ζ-regularization. We will see that the resulting value coincides with the usual *Fuglede-Kadison determinant* of A.
If $\mathcal{H}$ is not finitely generated, then there is no feasible method to extend to the determinant to any reasonable class of *bounded* operators. However, we will see that in this case, there exists a very natural class of *unbounded* operators over $\mathcal{H}$ that still admit a ζ-regularized determinant.
These techniques can be used to define both the *analytic* and the *combinatorial torsion*, associated to a smooth, closed manifold M and a representation ρ of its fundamental group. If time permits, we will eventually state some important results relating these two values.