

L^2 -invariants

(Lecture course, winter term 2015/16)

Description

Classical algebraic topology is concerned with invariants such as Betti numbers, Euler characteristic and Reidemeister torsion of compact spaces. It is a fruitful idea to try and find counterparts to these invariants on the universal covering. The latter, however, is no longer compact unless the fundamental group is finite. Technically, this has the effect that common associated algebraic structures, like homology and chain complexes, might be infinite-dimensional and do not allow an ad hoc definition of useful invariants.

The remedy to this dilemma is passing to the “ L^2 -completion” to obtain Hilbert spaces. For these objects powerful tools from analysis give a means to define L^2 -invariants which extract valuable and otherwise invisible information. L^2 -invariants form an area of active research and have proven their usefulness in contexts as diverse as group theory, differential geometry, ergodic theory, K -theory and more recently also knot theory and quantum groups.

Contents

- Hilbert modules and von Neumann dimension
- L^2 -Betti numbers of CW complexes and groups
- Novikov–Shubin invariants
- Fuglede–Kadison determinant and L^2 -torsion

Recommendations

Concepts from “Introduction to Geometry and Topology” (fundamental group and covering spaces) as well as “Algebraic Topology” (CW complexes, chain complexes, cellular homology) will be used. I intend to explain methods from other fields when needed.

Schedule

- Lecture: Thu, 15:45–17:15, 20.30 SR 2.58
- Exercise class: Tue, 09:45–11:15, 20.30 SR 2.58

References

W. Lück, *L^2 -invariants: theory and applications to geometry and K -theory*, Erg. Math. Grenzgeb. (3), vol. 44, Springer-Verlag, Berlin, 2002