

GGT Seminar: Profinite rigidity

Talk 1 (Profinite rigidity and two simple examples). Show that finitely generated abelian groups are profinitely rigid, the first Betti number is a profinite invariant [Rei18, Section 3] and treat Baumslag’s example: meta-cyclic groups are not profinitely rigid [Bau74].

Talk 2 (ℓ^2 -Betti numbers). Show that the first ℓ^2 -Betti number is a profinite invariant. Show that higher ℓ^2 -Betti numbers are in general not profinite invariants. [Stu18] [Rei13]

Talk 3 (Amenability). Sketch the proof that amenability is *not* a profinite invariant. [KS23]

Talk 4 (The Figure-eight Knot). The figure-eight knot complement is profinitely rigid among 3-manifold groups [BR20]. Sketch a proof for this result.

Talk 5 (3-manifolds). Give an overview over profinite properties of 3-manifolds. [Rei18, Section 4] In particular, introduce the characterisation of hyperbolic groups among 3-manifolds. [WZ16]

Talk 6 (More 3-manifolds). Sketch the main ingredients in the proof of the following result: finite-volume hyperbolic 3-manifolds are almost profinitely rigid. [Liu23, Theorem 1.1]

Talk 7 (Cohomology). Introduce the notion of cohomological goodness [Ser01]. Survey the cohomological properties of profinite groups and present the proof that Bianchi groups are good. [GJZ08].

Talk 8 (Property (T)). Recall Property (T) and show that it is not profinite. [Aka10]

References

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