Minisymposium „Topology and geometry of Lie group actions“

The concept of symmetry has always played a crucial role in understanding geometric objects. A classical way of modeling symmetry of spaces is to impose the action of a group. This approach still proves to be very successful in modern mathematics yielding beautiful results in a multitude of fields, such as algebraic and geometric topology, or Riemannian and symplectic geometry.

In this mini-symposium we will discuss recent results in this area; on the one hand concerning the theory of Lie transformation groups itself, and on the other hand using the existence of Lie group actions as a means to understand various geometric structures.

Abstracts

22.9., 10.30, Isometric Lie group actions on Alexandrov spaces (Fernando Galaz-García).
Alexandrov spaces (with curvature bounded below) are a natural synthetic generalization of Riemannian manifolds. In this talk I will discuss recent developments on the geometry and topology of Alexandrov spaces with isometric actions of compact Lie groups.

22.9., 11.10, Recent results on polar actions (Andreas Kollross).
Isometric Lie group actions on Riemannian manifolds are called polar if there is a submanifold, called section, which meets all orbits of the group action and meets them orthogonally at any intersection point; they are called hyperpolar in the special case where the section is flat.

I will talk about a result on hyperpolar actions on reducible compact symmetric spaces and a recent classification of infinitesimally polar (i.e. all slice representations are polar) actions on compact rank one symmetric spaces. The latter is joint work with Claudio Gorodski.

22.9., 11.50, Congruence invariants for holomorphic maps and rigidity (Stefan Bechtluft-Sachs).
Metric rigidity of holomorphic maps (as that of smooth maps between Riemannian mannifolds) generally requires some kind of non degeneracy assumptions. Thus holomorphic maps in complex projective spaces are congruent if they have the same first fundamental form. In Hermitian symmetric target spaces of higher rank however, the maps should be full in the sense that their osculating space exhausts the ambient tangent space.

In Grassmannians this can be resolved by fixing the second fundamental form as well, but this over determines the map. For holomorphic maps into Grassmannians, we determine a complete set of invariants and some of the arising relations. Most of this also works for harmonic maps.

23.9., 10.30, Tamed symplectic structures on solvmanifolds (Anna Fino).
Symplectic forms taming complex structures on compact manifolds are strictly related to a special type of Hermitian metrics, known in the literature also as pluriclosedmetrics. I will present some general results on pluriclosedmetrics and their link with symplectic geometry for solvmanifolds. Moreover, I will show for certain 4-dimensional non-Kaehler 4-manifolds some recent results about the Calabi-Yau equation in the context of symplectic geometry.

23.9., 11.10, Localization for K-contact manifolds (Jonathan Fisher).
The Jeffrey-Kirwan residue formula computes the intersection pairings on a symplectic quotient $M//G$ as the residues of certain meromorphic differential forms associated to the fixed point set M^T, where T is a maximal torus of the compact Lie group G. Key ingredients of the proof are equivariant integration and localization. We extend these techniques to the setting of K-contact manifolds and obtain an analogous residue formula. This is based on joint work with Lana Casselmann.

23.9., 11.50, Group actions in symplectic geometry (Silvia Sabatini).
Let a torus act on a compact symplectic manifold with isolated fixed points. In this talk I will discuss about recent results concerning the classification of the topological invariants of such a manifold, including equations involving the Chern numbers of the manifold, depending on its minimal Chern number. This includes both the Hamiltonian and non-Hamiltonian case.

24.9., 10.30, Assignments for topological group actions (Liviu Mare).
Let T be a (compact) torus that acts on a topological space X. A polynomial assignment is a map A that assigns to any $x \in X$ a polynomial function $A(x) : \mathrm{Lie}(T_x) \to \mathbb{R}$, where T_x is the stabilizer of x; A is required to be T-invariant and satisfy a certain compatibility condition involving fixed points of various subtori of T. The space of all such assignments is an algebra over the polynomial ring of $\mathrm{Lie}(T)$, the so-called *assignment algebra*. This notion was introduced by Ginzburg, Guillemin, and Karshon in 1999. For smooth actions on

manifolds, connections with the T-equivariant cohomology algebra were established recently by Guillemin, Sabatini, and Zara (2014). I will explain that the same relationship between assignments and equivariant cohomology exists in the topological setting. I will also discuss assignment versions of the Chang-Skjelbred lemma and the Goresky-Kottwitz-MacPherson presentation. This is a report on joint work with Oliver Goertsches (LMU Munich).

24.9., 11.10, Weakly complex homogeneous spaces (Uwe Semmelmann).
We present a classification of compact homogeneous spaces with positive Euler characteristic admitting an almost complex structure and more generally a tangent bundle which is stably complex. We show that such a space is a product of compact equal rank homogeneous spaces which either carry an invariant almost complex structure, or have stably trivial tangent bundle, or belong to an explicit list of weakly complex spaces which have neither stably trivial tangent bundle, nor carry invariant almost complex structures. The talk is based on joint work with Andrei Moroianu.

Kontakt:
Manuel Amann, manuel.amann@kit.edu
Oliver Goertsches, goertsches@math.lmu.de