

# Workshop on profinite rigidity

**Organisers:** Benjamin Brück (Universität Münster), Jerónimo García-Mejía (KIT Karlsruhe), Kevin Klinge (KIT Karlsruhe), Matthias Uschold (Universität Regensburg)

**Location:** The workshop takes place at the maths faculty of the KIT Karlsruhe (see <https://www.math.kit.edu/fakmath/page/anreise/> for directions. The talks will be held in room 1.067, the coffee breaks will take place in room 1.058 (both on the first floor<sup>1</sup>)

## Monday, August 26th

9:00–10:00	<b>Registration</b>	
10:00–10:45	<b>Lukas Schneider</b> KIT Karlsruhe	Profinite rigidity - two examples
10:45–11:15	<b>Coffee break</b>	
11:15–12:15	<b>Jonathan Fruchter</b> Universität Bonn	Lecture 1
12:30–13:30	<b>Lunch</b>	
14:00–14:45	<b>Sruthy Joseph</b> Universität Münster	$\ell^2$ -Betti numbers
14:45–15:30	<b>Coffee break</b>	
15:30–16:15	<b>Giada Serafini</b> HHU Düsseldorf	Profinite properties of $\ell^2$ -invariants

## Tuesday, August 27th

09:30–10:30	<b>Jonathan Fruchter</b> Universität Bonn	Lecture 2
10:30–11:15	<b>Coffee break</b>	
11:15–12:00	<b>Franziska Hofmann</b> Universität Regensburg	Amenability
12:30–13:30	<b>Lunch</b>	
14:00–14:45	<b>Raquel Murat</b> Universität Münster	Cohomology
14:45–15:30	<b>Coffee break</b>	
15:30–16:15	<b>Philip Möller</b> HHU Düsseldorf	Profinite rigidity and flexibility of Coxeter groups
16:30–17:15	<b>Marco Amelio</b> Universität Münster	Property (T)

<sup>1</sup>in the European sense, i.e. not the ground floor

## Wednesday, August 28th

09:30–10:30	<b>Jonathan Fruchter</b> Universität Bonn	Lecture 3
10:30–11:15	<b>Coffee break</b>	
11:15–12:00	<b>Adrian Baumann</b> HHU Düsseldorf	The figure eight knot complement
12:30–13:30	<b>Lunch</b>	

## Abstracts

### **Jonathan Fruchter (Universität Bonn):** Minicourse (3 parts)

We will give a general overview of the field of profinite rigidity, with an emphasis on methods coming from geometric group theory. In the first talk, we will mention some of the main results and recent breakthroughs in the field, and get a taste of some of the proofs. The second talk will revolve around Remealennikov's question (is a non-abelian free group profinitely rigid?). Lastly, in the third talk, we will introduce the profinite version of Bass-Serre theory and see some of its applications.

### **Lukas Schneider (KIT):** Profinite rigidity - two examples

For a group  $\Gamma$ , let  $\widehat{\Gamma}$  denote the profinite completion of  $\Gamma$ . Profinite rigidity is closely related to the following problem: Let  $\Gamma, A$  be groups with the same profinite completion i.e.  $\widehat{\Gamma} \cong \widehat{A}$ . Can one conclude  $\Gamma \cong A$ ? In general the answer is no. If the answer is yes we will call  $\Gamma$  profinitely rigid. We will first prove that finitely generated abelian groups are profinitely rigid. As a consequence we get that the first Betti number of a group is a profinite invariant. Following a paper by Baumslag we will construct metacyclic groups which are not profinitely rigid.

### **Sruthy Joseph (Universität Münster):** $\ell^2$ -Betti numbers

Show that the first  $\ell^2$ -Betti number is a profinite invariant. Show that higher  $\ell^2$ -Betti numbers are in general not profinite invariants.

### **Giada Serafini (HHU Düsseldorf):** Profinite properties of $\ell^2$ -invariants

Although higher  $\ell^2$ -Betti numbers are in general not profinite invariants, some interesting information on the  $\ell^2$ -cohomology of  $S$ -arithmetic groups still seems to be preserved under profinite commensurability. This leads us to prove a stability result for another  $\ell^2$ -invariant: the sign of the Euler characteristic. We will discuss the key steps needed to prove such a result in terms of local-global principles in Galois cohomology and ask if this technique can be extended to similar open problems.

### **Franziska Hofmann (Universität Regensburg):** Amenability

Sketch the proof that amenability is not a profinite invariant.

### **Raquel Murat (Universität Münster):** Cohomology

Introduce the notion of cohomological goodness. Survey the cohomological properties of profinite groups and present the proof that Bianchi groups are good.

### **Philip Möller (HHU Düsseldorf):** Profinite rigidity and flexibility of Coxeter groups

Coxeter groups are abstract reflection groups introduced in 1933. They have been studied from different points of view ever since and are commonly studied in geometric group theory. In this

walk I want to discuss them with regard to profinite rigidity, i.e. to what extent they are determined by their finite quotients. This is based on joint and ongoing work with Sam Corson, Sam Hughes and Olga Varghese.

**Marco Amelio (Universität Münster):** Property (T)  
Recall Property (T) and show that it is not profinite.

**Adrian Baumann (HHU Düsseldorf):** The figure eight knot complement  
The figure-eight knot complement is profinitely rigid among 3-manifold groups. Sketch a proof for this result.